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Quantum-like Modeling: from Economics to Social Laser

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Abstract

This is a brief review of the recent development of quantum-like modeling, in decision making, finance, social and political science. We point to the recent quantum information revolution elevating the role of the information dimension of quantum theory. Then the essentials of quantum probability are briefly presented with emphasis on the formula of total probability with the interference term as a tool for amplification of probability (as is done in quantum computing). We stress that this amplification is the essence of the quantum probability update, non-Bayesian update. We show that contextuality is the basic reason for the applicability of quantum probability to modeling of behavior of human agents - the quantum-like paradigm. We then discuss semiclassical modeling in economics and finance. Finally, we present the model of "social laser" for stimulated amplification of social actions (SASA). This is a new field of research. We formulate a number of open problems, both in theory and social engineering. One of the main intriguing problems is the mathematical formalization of the notion of social energy (with its possible generalizations such as, e.g., financial energy).

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1 INTRODUCTION

Nowadays quantum-like models are widely used in psychology, decision making, economics, finance, social and political science, game theory, and in the modeling of the behavior of macroscopic biological systems such as cells or proteins (see, e.g., [1,3,15,16,22,23] and references in these monographs).

In decision making (see again [1, 3, 15, 16, 22, 23]) important advances were made on how to better formalize decision making behavior (especially within the context of paradoxical decision making outcomes). A central aspect in that research, is the idea of so called 'contextual interaction'.

In economics and finance, more recent work is to be noticed on essentially using the semi-classical approach to quantum mechanics. This approach was already conceived in [24], more than 20 years ago and was recently applied to simple financial data in [36], [5]. We also point to intensive theoretical modeling of asset price dynamics on the basis of Bohmian mechanics [6, 7, 13, 14].

The quantum-formalism has also found applications in other venues such as political science where explicit use is made of so called open systems [26]. In game theory we also see another area of work [27] where the celebrated Aumann theorem is put into a quantum-like setting with surprisingly interesting results.

In all of the work which applies formalisms of quantum mechanics to areas outside of its natural remit, it is absolutely essential to stress that none of this work implies that macroscopic events are quantum mechanical.

The theory of social laser [28–32, 37], a quantum-like model of *Stimulated Amplification of Social Actions (SASA),* is one of the recent fruits of quantum-like modeling in applications to psychosocial processes. The basics of this theory will be presented in section 6.

2 QUANTUM INFORMATION REVOLUTION AND QUANTUM-LIKE MODELING

We start by describing the present state of affairs in quantum theory and argue how this impacts applications outside of physics, i.e. so-called *quantum-like modeling.* The recent *quantum information revolution* has impact not only on the development of quantum theory, experiment, and technology, but also on quantum foundations. The most fundamental problems of quantum theory were enlightened from another angle. For quantum-like modeling, the main consequence of the quantum information revolution is that nowadays quantum physical systems are widely treated as just carriers of information. Hence, quantum physics is rapidly transformed into quantum information theory. Physical properties of carriers of quantum information are often completely ignored. One is interested merely in the features of information processing per sé. In particular, physical space-time has practically disappeared from quantum (information) theory. As a consequence, the majority of experts in quantum information theory know very little about real quantum physics. The quantum scenarios are considered in finite dimensional state spaces. Linear algebra became the main mathematical tool, instead of functional analysis which had been playing the crucial role during the first seventy years of the development of quantum mechanics.

Nowadays one is not so much interested in the physical properties of the *transformers of quantum information*. It is natural to search for other classes of information transformers, which are different from genuine quantum physical systems, but transform information in a quantum-like manner. The latter means that we can use the basic tools of the quantum formalism to model the transformer's behavior. The most basic tool is the quantum probabilistic formalism, the calculus of complex probability amplitudes, endowed with *Born's rule* coupling complex amplitudes with probabilities. Another powerful mathematical tool is the use of *tensor products* to model (information) behavior of compound systems - a group of transformers interacting with information fields. Such interactions are modelled with the aid of the *theory of open quantum systems.*

Surprisingly to find quantum-like systems, we need not go deeply into the microworld, or isolate systems from environments, or freeze them to absolute zero. Anybody has such a system inside him/herself: our brain. The analogy between the behavior of genuine quantum physical systems such as electrons or photons, and the psychological behavior of humans has been discussed, since the early days of quantum mechanics. We can mention, e.g., the Pauli-Jung [18] correspondence, see, e.g., one of the letters of Jung to Pauli:

"As the phenomenal world is an aggregate of the processes of atomic magnitude, it is naturally of the greatest importance to find out whether, and if so how, the photons (shall we say) enable us to gain a definite knowledge of the reality underlying the mediative energy processes. Light and matter both behave like separate particles and also like waves. This ... obliged us to abandon, on the plane of atomic magnitudes, a causal description of nature in the ordinary space-time system, and in its place to set up invisible fields of probability in multidimensional spaces."

However, only in the present quantum information era, has this analogy started to be formalized on the basis of quantum theory.

We need to remark that quantum-like modeling has no direct coupling to the quantum physical brain project: an attempt to reduce cognition and consciousness to genuine quantum physical processes in the brain (Umezawa, Hameroff, Penrose, Vitiello, see, e.g., [17, 34, 38–40]). At the same time, we do not criticize this project, we adopt a neutral position. In any event, the quantum-like model for information processing by humans and, in particular stimulated amplification of social actions (SASA), are not based on the assumption that quantum physics plays some role in human cognition. Moreover, the quantum-like representation of information by the brain can be constructed on the basis of classical electrochemical processes in neural networks and uncertainty in the generation of action potentials [25]. However, the latter is not a topic of this paper .

3 QUANTUM PROBABILITY AS THE BASIS OF QUANTUM-LIKE MODELING

We recall that as early as the 1970s, Tversky and Kahneman (Nobel prize in economics in 2002, for prospect theory, which he co-developed with Tversky) have been demonstrating cases, where the classical probabilistic (CP) prescription used in modelling human behavior persistently diverges from what is observed, see, for example, [19–21]. Their recommendation was to abandon CP and proceed with heuristics in decision making theory. However, we can point to another way of the resolution of the aforementioned problem. This is the use of nonclassical probabilistic models and, especially, quantum probability (QP). The latter is one of the most developed nonclassical models. Here we present briefly its essentials, in a non-physicist gentle manner.

3.1 Born's Rule

CP was mathematically formalized by Kolmogorov (1933) [33]. This is the calculus of probability measures, where a non-negative weight $p(A)$ is assigned to any event A.

The main property of CP is its additivity: if two events A_1, A_2 are disjoint, then the probability of disjunction of these events equals to the sum of probabilities:

$$P(A_1 \vee A_2) = P(A_1) + P(A_2).$$

Quantum probability (QP) is the calculus of complex amplitudes or in the abstract formalism complex vectors. Thus, instead of operations on probability measures one operates with vectors. We can say that QP is a *vector model of probabilistic reasoning.* Each complex amplitude ψ gives the probability by the Born's rule: *Probability is obtained as the square of the absolute value of the complex amplitude.*

$$p = |\psi|^2.$$

3.2 Interference of Probabilities

By operating with complex probability amplitudes, instead of the direct operation with probabilities, one can violate the basic laws of CP, in particular, additivity of probability. One can get that, for disjoint events, the probability of disjunction is strictly smaller or larger than the sum of probabilities:

$$P(A_1 \vee A_2) < P(A_1) + P(A_2)$$

or

$$P(A_1 \vee A_2) > P(A_1) + P(A_2),$$

since QP calculus leads to the formula

$$P(A_1 \vee A_2) = P(A_1) + P(A_2) \\ +2\cos\theta\sqrt{P(A_1)P(A_2)}. \quad (1)$$

The additional term is known as the interference term. We recall that interference is the basic feature of waves, so often one speaks about probability waves.

3.3 Non-Bayesian Probability Inference

Nowadays, QP is widely used to model cognition, psychology, decision making as well as economics and finance. QP provides the possibility to relax some constraints on statistical data posed by CP. One of such constraints is the classical *formula of total probability* (FTP). It is a consequence of the additivity of CP and the Bayes formula in the definition of conditional probability, namely,

$$P(B|A) = \frac{P(B \cap A)}{P(A)},\ P(A) > 0. \quad (2)$$

Consider the pair, a and b, of discrete classical random variables. Then

$$P(b = \beta) = \sum_{\alpha} P(a = \alpha) P(b = \beta | a = \alpha). \quad (3)$$

Thus the b-probability distribution can be calculated from the a-probability distribution and the conditional probabilities $P(b = \beta | a = \alpha)$.

We remark that the Bayesian approach to conditional probability plays a crucial role in CP when it models cognition, machine learning, AI, especially in applications of Bayesian networks. Thus, the QP-modeling can be considered as the extension of the boundaries of Bayesian probability inference. In QP, formula (1) leads to the perturbation of classical FTP known as FTP with the interference term [23]:

$$P(b = \beta) = \sum_{\alpha} P(a = \alpha) P(b = \beta | a = \alpha) + \\ +2 \sum_{\alpha_1 < \alpha_2} \cos\theta_{\alpha_1 \alpha_2} \sqrt{P(a = \alpha_1) P(b = \beta | a = \alpha_1) P(a = \alpha_2) P(b = \beta | a = \alpha_2)} \quad (4)$$

If the interference term is positive, then the QP-calculus would generate a probability that is larger than its CP-counterpart given by the classical FTP (4). In particular, this probability amplification is the basis of the quantum computing supremacy.

There is a plenty of statistical data from cognitive psychology, social and political science, finance, and game theory demonstrating that humans use this amplification and operate with nonclassical probability updates [12].

3.4 Quantum-like Paradigm: Contextuality

The following paradigm can be used to motivate the applications of QP outside of physics.

Quantum-like paradigm ([24], see also monograph [23] for details):

The mathematical formalism of quantum information and probability theories can be used to model behavior not only of genuine quantum physical systems, but all *context-sensitive systems,* e.g., humans or AI-systems.

We emphasize the following consequences of this paradigm:

- Contextual information processing cannot be based on complete resolution of ambiguity.
- It is meaningless to do this for a concrete context, if tomorrow's context will be totally different.
- Therefore such systems process

ambiguities, and process superpositions of alternatives.

4 SEMICLASSICAL MODELING IN ECONOMICS AND FINANCE

As was already mentioned, in economics and finance, the recent progress was essentially based on the semi-classical approach to quantum mechanics. The propensity for augmented insights with the use of the semi-classical approach centers around two key questions:

- Can we dig deeper in classifying public information within different types of information?
- Can the augmented second law of Newton, give us price paths which are influenced by changes of information (which are themselves fueled by the so called quantum potential)?

We do see, within a purely physics context, at this point in time some fluid dynamical interpretations [4] of the semi-classical approach which are very interesting and which may, one day, hold promise for applications outside of physics.

5 QUANTUM-LIKE MODELING OF SOCIO-POLITICAL PROCESSES

Recent years have been characterized by stormy social protests throughout the world. By using an analogy with cosmology, we can say that the collapse of the Soviet Union was the singularity point for the Social Big Bang. These protests have some commonalities, but at the same time their socio-political, psychological, and economic contexts show essential differences. An important class of such protests is labeled as *color revolutions*. Such events are often characterized by brutal clashes and lead to civil wars. Other social forms of social protests take place inside democratic societies. We can mention Brexit and the election of Donald Trump as the president of the USA. Although the latter social actions are not brutal, their internal and global socio-political impact is maybe even higher than the impact of some color revolutions.

The analysis of these events in the social and political literature, is characterized by a huge diversity of opinions. This is not surprising, because typically each author emphasizes some concrete feature of a social protest's context. Such socio-political biases lead to a variety of explanations for the recent world-wide waves of massively coherent social actions.

In contrast to experts in the social and political sciences, we are not looking for the concrete social-political (or economic, or financial, or historical) roots of protests. We want to find the general conditions leading to the generation of waves of social actions and generally we ignore the concrete social, political, economic, financial, and historical variables involved in an event. This paper is neither on sociology nor is it on politics. It is about the quantum-like operational modeling of socio-political processes [15, 26]

How can one model some process without going deeply into details, i.e.

without trying to determine and quantify the complex variables involved in the process? We remark that in socio-political processes such variables are so complex that typically it is meaningless to even try to determine them (whether quantitatively or even qualitatively). Those who know (or at least have heard of) the basics of quantum mechanics and its foundational problems, can immediately recognize that this problem of social variables is similar to the problem of hidden variables in quantum mechanics. In the latter, one is not able to determine variables permitting a finer state representation than given by the wave function (complex probability amplitude).

In passing, we may want to say that the semi-classical approach, we mentioned in the introduction and section 4 of this paper, actually revived hidden variables. A central concept in this theory is the idea of the so called quantum potential (mentioned in the former section). It is debatable to what type of energy such potential refers to, but what it is quite clear is that this type of potential has a proportional relationship with so called Fisher information [35] and that type of information has also a link to the well known Cramer-Rao bound used in econometrics. Besides the thorny foundational issues which are called to the foreground with the use of hidden variables, there is area of fraught-upon work (so to speak), but which opens up interesting avenues and that is non-Hermiticity. In finance, for instance, the Black-Scholes (financial option pricing) Hamiltonian is non-Hermitian. There is much recent work on this, in areas completely outside of social science [2]. Basic features such as the preservation of probability and an augmented understanding in frameworks where open systems would be natural to use (such as in political science) can be obtained with the use of the non-Hermitian Hamiltonian version of the Schrödinger equation (see also [16]).

In this paper we are not interested in the problem of the impossibility to use hidden variables for quantum processes. As we mentioned already, this is a complicated problem characterized by a diversity of opinions and stormy debates. We feel comfortable by recognizing that the quantum formalism provides an excellent description of observations which match very well with experiment. We want to proceed in the same way by modeling socio-political processes: to model observational outputs of social perturbations, but without attempting to find the "hidden variables" behind them.

6 STIMULATED AMPLIFICATION OF SOCIAL ACTIONS: "SOCIAL LASER"

We remark that the socio-political perturbations under consideration (section 5) are characterized by the *cascade dynamics* leading to the exponential amplification of coherent social actions. In quantum physics, such exponential and coherent amplification is the basic feature of a laser's functioning. 'Laser' is the acronym for *Light Amplification by Stimulated Emission of Radiation*. In this paper we explore (following papers [28–32, 37]) the theory of laser to model aforementioned waves of social protests, from color revolutions to Brexit and Trump's election. We call such social processes *Stimulated Amplification of Social Actions* (SASA), but to keep closer to the analogy with physics, we shall merely operate with the terminology *"social laser".* Our main efforts are directed to distill laser's theory from optics (from light (i.e, we shall struggle against the letter "L" in "laser")). Instead of the standard operating with frequencies or wave-lengths, we shall operate with *social energy.* In physics, the energy and frequency are coupled by a simple formula. But in the social and political sciences, the notion of frequency seems to be ambiguous and we keep to the energy representation.

The basic component of physical laser is the gain medium, an ensemble of atoms. Energy is pumped into this medium aimed to approach the state of population inversion, i.e, the state where more than 50% of atoms are excited. Then a coherent bunch of photons is injected into the gain medium and this bunch stimulates the cascade process of emission of the coherent photon-beam. If the power of pumping is very high, i.e., it is higher than the so-called lasing threshold, all energy of pumping is transferred into the output beam of coherent radiation. To make this beam essentially stronger, the laser is equipped by an additional component, the laser's resonator (typically in the form of optical cavity). The laser's resonator also improves coherence of the output beam, by eliminating from the beam, photons that were generated via spontaneous emission in the gain medium.

The notion of social energy is the main novel component of our quantum-like modeling. To justify the use of the social analog of physical energy, we use the quantum-mechanical interpretation of energy, not as an internal feature of a system, but as an observable quantity. Thus, like the case of an electron, we cannot assign to a human individual the concrete value of social energy. There are mental states in superposition of a few different values of social energy. However, by designing proper measurement procedures we can measure human's energy.

An important comment should be made on the first letter in SASA. Typically 'stimulated' is associated with a kind of external ignition of amplification of social actions. We remind that in physics 'stimulation' means that under the influence of the electromagnetic radiation, the laser's gain medium composed of say atoms starts to emit photons carrying the same characteristics as photons in the field (coherence). Such cascade generating photons need not come from an external source, they can be generated in the gain medium by the process of spontaneous emission. The power of such a field (photons' flow) increases exponentially with the aid of the *laser's resonator.* In physics, this is typically an optical cavity.

Similarly, the spontaneous stimulation of emission can start in a social gain medium which approaches the population inversion. The latter means the number of humans, social analogs of atoms (s-atoms), occupying the excited state is higher than the number of humans occupying the non-excited state. In the same way, as in physics, the social lasers have to be equipped with resonators, e.g., in the form of *Echo Chambers* which are typically Internet based.

On the other hand, similarly to physics, 'stimulation' of social lasing can be also generated by the external information field, a portion of information communications injected in a human gain medium with population inversion. Social resonators also play the crucial role in the generation of lasing.

The word 'stimulated' is referred to both forms of ignition of lasing:

- by a portion of spontaneously emitted messages,
- by a portion of externally injected messages.

For social engineering, the latter way of ignition is preferable.

As is well known, the analogy with the physical laser played a fundamental role in establishing the basic principles of *synergetics* [8–10]. Haken [11] emphasized that the laser can be considered as *trailblazer of synergetics.* He also pointed to "analogies between the behavior of a laser threshold and phase transition" and to the "significance of the laser as a prototype of systems which produce spatial or temporal structures of self-organization", Haken [11]. This viewpoint is very supportive for the present project on social lasing.

At the same time, we have to point to the crucial difference between quantum information modeling of social lasing presented in this paper, and the use of the analogy with the physical laser to motivate the development of the science about cooperation - synergetics. Haken's idea was to use this analogy to unify in one framework, a huge variety of cooperation processes. In such a project the quantum information basis of the laser's functioning was not and, moreover, should not be emphasized. The majority of "laser-like" synergetic processes have no relation to the quantum structure of information processing in the social systems. In particular, Haken [11] considered classical laser-like equations to illustrate the mathematical analogy with self-organization processes in classical physics. Our aim is different. We want to formalize quantum information features of social, and more generally, information systems which can lead to SASA.

The model developed in the series of works [28–32, 37] is the first step towards social laser engineering. For the moment, we derived only the most fundamental constraints onto human gain media, functioning of social resonators, and information fields leading to the possibility of successful social lasing. The social laser project (as well as the physical laser project) is very complex and a lot of work is needed before we approach the level of theory, measurement, and experiment as in the physical project. We list the main problems which have to be resolved:

1. To develop a comprehensive theory of social resonators (analogs of optical cavities); in this paper we present only a schematic description.

2. To continue the analysis of constraints on a human gain medium responsible for social lasing.
3. To study the role of information overload in switching humans into an indistinguishability regime of information processing in that communications content is practically ignored.
4. To analyze the impact of elimination of social diversity (in culture, life style, food, nationality, gender, education, science, and economics) to prepare a proper human gain medium.
5. To elaborate and test measurement procedures for social energy and temperature.
6. To continue the determination of the basic parameters for social lasing, see papers [31, 32].

At the same time it is clear that in academic laboratories experimenting in social lasing is practically impossible. However, this should not be considered as a big problem of the social laser project. Nowadays, the intensity of social lasing within the international arena approaches a high level, and scientists can enjoy this great possibility for testing their models.

For the moment, social laser's theory provides merely a qualitative characterization of basic features of human gain mediums and information fields generated by the mass-media leading to successful social lasing. In article [32], we made the first steps towards quantitative predictions.

This paper is oriented to scientists (as well as students) from the humanities and the social sciences, who are interested in the application of the social laser model and generally quantum-like models. As was emphasized, the quantum information revolution minimized the genuine physical content of quantum theory. The mathematical content also was essentially simplified. Partially, this paper is also oriented to physicists who are interested in the applications of quantum physics and thermodynamics in social science.

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